Typeless Programming in Java 5 and 7

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20. September 2010

Overview

Introduction

Extensions of the Java type–system

- parametrized types, type variables, type terms, wildcards
  e.g.

  `Vector<? extends AbstractList<? super Integer>>`

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- Sometimes principal types in Java are *intersection types*, which are not expressible (contradictive of writing re-usable code)

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- Often it is not obvious, which are the *best* types for methods and variables
- Sometimes principal types in Java are *intersection types*, which are not expressible (contradictive of writing re-usable code)

$\Longrightarrow$ Developing a type–inference–system, which determines principal types

Example: Multiplication of matrices

```
class Matrix extends Vector<Vector<Integer>> {
    Matrix mul(Matrix m) {
        Matrix ret = new Matrix();
        int i = 0;
        while(i <size()) {
            Vector<Integer> v1 = this.elementAt(i);
            Vector<Integer> v2 = new Vector<Integer>();
            int j = 0;
            while(j < size()) {
                int erg = 0;
                int k = 0;
                while(k < v1.size()) {
                    erg = erg + v1.elementAt(k)
                        * m.elementAt(k).elementAt(j); k++; }
                v2.addElement(new Integer(erg)); j++; }
            ret.addElement(v2); i++; }
        return ret; }}
```

Alternative Typing

```
class Matrix extends Vector<Vector<Integer>> {
    Matrix/Vector<Vector<Integer>> mul(Matrix/Vector<Vector<Integer>> m) {
        Matrix/Vector<Vector<Integer>> ret = new Matrix();
        int i = 0;
        while(i <size()) {
            Vector<Integer> v1 = this.elementAt(i);
            Vector<Integer> v2 = new Vector<Integer>();
            int j = 0;
            while(j < size()) {
                int erg = 0;
                int k = 0;
                while(k < v1.size()) {
                    erg = erg + v1.elementAt(k)
                        * m.elementAt(k).elementAt(j); k++; }
                v2.addElement(new Integer(erg)); j++; }
            ret.addElement(v2); i++; }
        return ret; }}
```

Purpose: Typless

```
class Matrix extends Vector<Vector<Integer>> {
    mul(m) {
        ret = new Matrix();
        i = 0;
        while(i <size()) {
            v1 = this.elementAt(i);
            v2 = new Vector<Integer>();
            j = 0;
            while(j < size()) {
                erg = 0;
                k = 0;
                while(k < v1.size()) {
                    erg = erg + v1.elementAt(k)
                        * m.elementAt(k).elementAt(j); k++; }
                v2.addElement(new Integer(erg)); j++; }
            ret.addElement(v2); i++; }
        return ret; }}
```

System determines the principal typing(s)

```
mul: Matrix → Matrix &
     Matrix → Vector<Vector<Integer>>
     & ... &
     Vector<? extends Vector<? extends Integer>>
                                  → Vector<? super Vector<Integer>>
```

Type inference algorithm for Java 5[2]

The Idea

---

[1] L. Damas, R. Milner. Principal type-schemes for functional programs.

[2] M. Plümicke. Typeless Programming in Java 5.0 with wildcards. PPPJ 2007

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Hindley/Milner Type inference [Damas, Milner 1982][1]

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$+$ function template $(ty_1 \times \ldots ty_n) \rightarrow ty_0$
(first–order functions)

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+ subtyping

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Hindley/Milner Type inference [Damas, Milner 1982][1]

– function type constructor → (no higher–order functions)

\+ function template $(ty_1 \times \ldots ty_n) \to ty_0$
(first–order functions)

\+ subtyping

\+ data and function polymorphism (overloading)

---

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Simple types $\mathrm{SType}_{TS}(\,BTV\,)$

- $BTV^{(ty)} \subseteq \mathrm{SType}_{TS}(\,BTV\,)$ (bounded type variables)
- $TC^{()} \subseteq \mathrm{SType}_{TS}(\,BTV\,)$ (0-ary type constructors/classes)
- For $ty_i \in \mathrm{SType}_{TS}(\,BTV\,)$

$$\begin{aligned} & \cup \{\, ? \,\} \\ & \cup \{\, ?\ \texttt{extends}\ \tau \mid \tau \in \mathrm{SType}_{TS}(\,BTV\,) \,\} \\ & \cup \{\, ?\ \texttt{super}\ \tau \mid \tau \in \mathrm{SType}_{TS}(\,BTV\,) \,\} \end{aligned}$$

and $C \in TC^{(a_1|_{b_1} \ldots a_n|_{b_n})}$ it holds

$$C\texttt{<}ty_1, \ldots, ty_n\texttt{>} \in \mathrm{SType}_{TS}(\,BTV\,)$$

if for $CC(\,C\texttt{<}ty_1, \ldots, ty_n\texttt{>}\,) = C\texttt{<}\overline{ty_1}, \ldots, \overline{ty_n}\texttt{>}$ holds:

$$\overline{ty_i} \leq^* b_i[a_j \mapsto \overline{ty_j} \mid 1 \leqslant j \leqslant n],$$

where

- $CC(\,\ldots\,)$ denotes the capture conversion
- $\leq^*$ is the subtyping ordering.

Abbreviation for wildcard–types

Instead of `A<? extends B>` we write

$$\mathtt{A<_{?}B>}$$

and instead of `C<? super D>` we write

$$\mathtt{C<^{?}D>}.$$

Subtyping ordering $\leq^*$

Reflexive and transitive closure of

- if θ `extends` θ' then $\underline{\theta \leq^* \theta'}$.
- if $\theta_1 \leq^* \theta_2$ then $\underline{\sigma_1(\theta_1) \leq^* \sigma_2(\theta_2)}$, where for each type variable a of θ_2 holds $\sigma_1(a) = \sigma_2(a) \in \mathrm{SType}_{TS}(BTV)$ (soundness condition).
- $\underline{a \leq^* \theta'}$ for $a \in BTV^{(\theta_1 \& \ldots \& \theta_n)}$ where $\exists \theta_i : \theta_i \leq^* \theta'$.
- It holds $\underline{C{<}\theta_1, \ldots, \theta_n{>} \leq^* C{<}\theta'_1, \ldots, \theta'_n{>}}$ if for θ_i and θ'_i either
    - $\theta_i = {}_?\overline{\theta}_i$, $\theta'_i = {}_?\overline{\theta}'_i$ and $\overline{\theta}_i \leq^* \overline{\theta}'_i$ or
    - $\theta_i = {}^?\overline{\theta}_i$, $\theta'_i = {}^?\overline{\theta}'_i$ and $\overline{\theta}'_i \leq^* \overline{\theta}_i$ or
    - θ_i, θ'_i are no wildcard arguments and $\theta_i = \theta'_i$ or
    - $\theta'_i = {}_?\theta_i$ or
    - $\theta'_i = {}^?\theta_i$
- From $C{<}\overline{\theta}_1, \ldots, \overline{\theta}_n{>} \leq^* C{<}\theta'_1, \ldots, \theta'_n{>}$ follows with $C{<}\overline{\theta}_1, \ldots, \overline{\theta}_n{>} = CC(\, C{<}\theta_1, \ldots, \theta_n{>}\,)$: $\underline{C{<}\theta_1, \ldots, \theta_n{>} \leq^* C{<}\theta'_1, \ldots, \theta'_n{>}}$
- $\underline{T|^{(\theta_1 \& \ldots \& \theta_n)} \leq^* \theta_i}$ for any $1 \leqslant i \leqslant n$.

Type unification [Pluemicke 2009][3]

Subtyping relation for type terms: $\leq^*$

Type Unification problem:

For two type terms θ_1 and θ_2 a substitution σ is demanded such that:

$$\sigma(\,\theta_1\,) \leq^* \sigma(\,\theta_2\,).$$

Base: Unification algorithm [Martelli, Montanari 1982][4]

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[3] M. Pluemicke. *Java type unification with wildcards*, INAP 07. LNAI 5437.

[4] A. Martelli, U. Montanari. An efficient unification algorithm.

Example

Subtyping relation:

$\texttt{Integer} \leq^* \texttt{Number}$

$\texttt{Stack<a>} \leq^* \texttt{Vector<a>} \leq^* \texttt{AbstractList<a>} \leq^* \texttt{List<a>}$

Example

Subtyping relation:

$$\texttt{Integer} \leq^* \texttt{Number}$$

$$\texttt{Stack<a>} \leq^* \texttt{Vector<a>} \leq^* \texttt{AbstractList<a>} \leq^* \texttt{List<a>}$$

Application of the algorithm:

$$\{ (\texttt{Stack<a>} \lessdot \texttt{Vector<}_{?}\texttt{Number>}), (\texttt{AbstractList<Integer>} \lessdot \texttt{List<a>}) \}$$

Example

Subtyping relation:

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Application of the algorithm:

$$\{\, (\mathtt{Stack<a>} \lessdot \mathtt{Vector<}{}_?\mathtt{Number>}), (\mathtt{AbstractList<Integer>} \lessdot \mathtt{List<a>}) \,\}$$

$$\Longrightarrow \{\, \mathtt{a} \lessdot_? {}_?\mathtt{Number},\ \mathtt{Integer} \lessdot_? \mathtt{a} \,\}$$

Example

Subtyping relation:

$$\texttt{Integer} \leq^* \texttt{Number}$$

$$\texttt{Stack<a>} \leq^* \texttt{Vector<a>} \leq^* \texttt{AbstractList<a>} \leq^* \texttt{List<a>}$$

Application of the algorithm:

$$\{\, (\texttt{Stack<a>} \lessdot \texttt{Vector<}{}_{?}\texttt{Number>}), (\texttt{AbstractList<Integer>} \lessdot \texttt{List<a>}) \,\}$$

$$\Longrightarrow \{\, \texttt{a} \lessdot_{?} {}_{?}\texttt{Number},\ \texttt{Integer} \lessdot_{?} \texttt{a} \,\}$$

$$\begin{aligned}
\Longrightarrow \{\, & \{\texttt{a} \doteq {}_{?}\texttt{Number}, \texttt{a} \doteq \texttt{Integer}\}, \{\texttt{a} \doteq {}_{?}\texttt{Number}, \texttt{a} \doteq {}_{?}\texttt{Number}\}, \\
& \{\texttt{a} \doteq {}_{?}\texttt{Number}, \texttt{a} \doteq {}_{?}\texttt{Integer}\}, \{\texttt{a} \doteq {}_{?}\texttt{Number}, \texttt{a} \doteq {}^{?}\texttt{Integer}\}, \\
& \{\texttt{a} \doteq \texttt{Number}, \texttt{a} \doteq \texttt{Integer}\}, \{\texttt{a} \doteq \texttt{Number}, \texttt{a} \doteq {}_{?}\texttt{Number}\}, \\
& \{\texttt{a} \doteq \texttt{Number}, \texttt{a} \doteq {}_{?}\texttt{Integer}\}, \{\texttt{a} \doteq \texttt{Number}, \texttt{a} \doteq {}^{?}\texttt{Integer}\}, \\
& \{\texttt{a} \doteq {}_{?}\texttt{Integer}, \texttt{a} \doteq \texttt{Integer}\}, \{\texttt{a} \doteq {}_{?}\texttt{Integer}, \texttt{a} \doteq {}_{?}\texttt{Number}\}, \\
& \{\texttt{a} \doteq {}_{?}\texttt{Integer}, \texttt{a} \doteq {}_{?}\texttt{Integer}\}, \{\texttt{a} \doteq {}_{?}\texttt{Integer}, \texttt{a} \doteq {}^{?}\texttt{Integer}\}, \\
& \{\texttt{a} \doteq \texttt{Integer}, \texttt{a} \doteq \texttt{Integer}\}, \{\texttt{a} \doteq \texttt{Integer}, \texttt{a} \doteq {}_{?}\texttt{Number}\}, \\
& \{\texttt{a} \doteq \texttt{Integer}, \texttt{a} \doteq {}_{?}\texttt{Integer}\}, \{\texttt{a} \doteq \texttt{Integer}, \texttt{a} \doteq {}^{?}\texttt{Integer}\} \,\}
\end{aligned}$$

Example cont.

$$\begin{array}{l}
\Longrightarrow \\
\{\, \{\, \mathtt{Integer} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq \mathtt{Integer} \,\}, \underline{\{\, {}_{?}\mathtt{Number} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}}, \\
\quad \{\, {}_{?}\mathtt{Integer} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}, \{\, {}^{?}\mathtt{Integer} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\}, \\
\quad \{\, \mathtt{Integer} \doteq \mathtt{Number}, \mathtt{a} \doteq \mathtt{Integer} \,\}, \{\, {}_{?}\mathtt{Number} \doteq \mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}, \\
\quad \{\, {}_{?}\mathtt{Integer} \doteq \mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}, \{\, {}^{?}\mathtt{Integer} \doteq \mathtt{Number}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\}, \\
\quad \{\, \mathtt{Integer} \doteq {}_{?}\mathtt{Integer}, \mathtt{a} \doteq \mathtt{Integer} \,\}, \{\, {}_{?}\mathtt{Number} \doteq {}_{?}\mathtt{Integer}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}, \\
\quad \underline{\{\, {}_{?}\mathtt{Integer} \doteq {}_{?}\mathtt{Integer}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}}, \\
\quad \{\, {}^{?}\mathtt{Integer} \doteq {}_{?}\mathtt{Integer}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\}, \\
\quad \underline{\{\, \mathtt{Integer} \doteq \mathtt{Integer}, \mathtt{a} \doteq \mathtt{Integer} \,\}}, \{\, {}_{?}\mathtt{Number} \doteq \mathtt{Integer}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}, \\
\quad \{\, {}_{?}\mathtt{Integer} \doteq \mathtt{Integer}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}\{\, {}^{?}\mathtt{Integer} \doteq \mathtt{Integer}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\} \,\}
\end{array}$$

Example cont.

$$\Longrightarrow$$

$$\begin{array}{l}
\{\, \{\, \mathtt{Integer} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq \mathtt{Integer} \,\}, \underline{\{\, {}_{?}\mathtt{Number} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}}, \\
\quad \{\, {}_{?}\mathtt{Integer} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}, \{\, {}^{?}\mathtt{Integer} \doteq {}_{?}\mathtt{Number}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\}, \\
\quad \{\, \mathtt{Integer} \doteq \mathtt{Number}, \mathtt{a} \doteq \mathtt{Integer} \,\}, \{\, {}_{?}\mathtt{Number} \doteq \mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}, \\
\quad \{\, {}_{?}\mathtt{Integer} \doteq \mathtt{Number}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}, \{\, {}^{?}\mathtt{Integer} \doteq \mathtt{Number}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\}, \\
\quad \{\, \mathtt{Integer} \doteq {}_{?}\mathtt{Integer}, \mathtt{a} \doteq \mathtt{Integer} \,\}, \{\, {}_{?}\mathtt{Number} \doteq {}_{?}\mathtt{Integer}, \mathtt{a} \doteq {}_{?}\mathtt{Number} \,\}, \\
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\quad \{\, {}_{?}\mathtt{Integer} \doteq \mathtt{Integer}, \mathtt{a} \doteq {}_{?}\mathtt{Integer} \,\}\{\, {}^{?}\mathtt{Integer} \doteq \mathtt{Integer}, \mathtt{a} \doteq {}^{?}\mathtt{Integer} \,\}\,\}
\end{array}$$

$$\Longrightarrow \{\, \{\, \mathtt{a} \mapsto {}_{?}\mathtt{Number} \,\}, \{\, \mathtt{a} \mapsto {}_{?}\mathtt{Integer} \,\}, \{\, \mathtt{a} \mapsto \mathtt{Integer} \,\}\,\}$$

The language Java$_{core}$

$$
\begin{array}{lll}
\mathit{Source} & := & \mathit{class}* \\
\mathit{class} & := & \text{Class}(\mathit{stype}, [\ \text{extends}(\ \mathit{stype}\ ),]\mathit{IVarDecl}*, \mathit{Method}*) \\
\mathit{IVarDecl} & := & \text{InstVarDecl}(\ \mathit{stype}, \mathit{var}\ ) \\
\mathit{Method} & := & \text{Method}(\ \mathit{mname}, [\mathit{stype}], (\mathit{var}[, \mathit{stype}])*, \mathit{block}\ ) \\
\mathit{block} & := & \text{Block}(\ \mathit{stmt}*\ ) \\
\mathit{stmt} & := & \mathit{block}\ \mid\ \text{Return}(\ \mathit{expr}\ )\ \mid\ \text{While}(\ \mathit{bexpr}, \mathit{block}\ ) \\
 & \mid & \text{LocalVarDecl}(\ \mathit{var}[, \mathit{stype}]\ )\ \mid \text{If}(\ \mathit{bexpr}, \mathit{block}[, \mathit{block}]\ ) \\
 & \mid & \mathit{stmtexpr} \\
\mathit{stmtexpr} & := & \text{Assign}(\ \mathit{var}, \mathit{expr}\ )\ \mid\ \text{New}(\ \mathit{stype}, \mathit{expr}*\ ) \\
 & \mid & \text{MethodCall}(\ [\mathit{expr},]\mathit{f}, \mathit{expr}*\ ) \\
\mathit{expr} & := & \mathit{stmtexpr}\ \mid\ \text{this}\ \mid\ \text{super} \\
 & \mid & \text{LocalOrFieldVar}(\ \mathit{var}\ )\ \mid\ \text{InstVar}(\ \mathit{expr}, \mathit{var}\ ) \\
 & \mid & \mathit{bexp}\ \mid\ \mathit{sexp}
\end{array}
$$

The algorithm

Type assumptions: For each absent type in the program a type-placeholder (fresh type variable) is assumed.

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New method type parameters: At the end remained type-placeholders are replaced by new introduced method type parameters.

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Intersection types: At the end each remained set of type assumptions forms one element of the result's intersection type.

Example: Multiplication of matrices: Type assumptions

```
class Matrix extends Vector<Vector<Integer>> {
   {α} mul({β} m) {
      {γ} ret = new Matrix();
      int i = 0;
      while(i <size()) {
         {ι} v1 = this.elementAt(i);
         {κ} v2 = new Vector<Integer>();
         int j = 0;
         while(j < size()) {
            {χ} erg = 0;
            int k = 0;
            while(k < v1.size()) {
               erg = erg + ({ξ}({ι} v1).elementAt(k))
                  * ({ψ}({φ} ({β} m).elementAt(k)).elementAt(j)); k++;}
            v2.addElement({χ} erg); j++; }
         ret.addElement({μ} v2); i++; }
      return ret; }}
```

ret = new Matrix ()

```
{$\alpha$} mul({$\beta$} m) {
    {$\gamma$} ret = {Matrix} new Matrix();
    ...
    return {$\gamma$} ret;
}
```

Unification: $\texttt{Matrix} \lessdot \gamma$

$\Rightarrow$

$$\begin{aligned}
\gamma &= \texttt{Matrix}\\
\gamma &= \texttt{Vector<Vector<Integer>>}\\
\gamma &= \texttt{Vector<}_{?}\texttt{Vector<Integer>>}\\
\gamma &= \texttt{Vector<}_{?}\texttt{Vector<}_{?}\texttt{Integer>>}\\
\gamma &= \texttt{Vector<}_{?}\texttt{Vector<}^{?}\texttt{Integer>>}\\
\gamma &= \texttt{Vector<}^{?}\texttt{Vector<Integer>>}
\end{aligned}$$

Type assumptions after the first unification

```
class Matrix extends Vector<Vector<Integer>> {
   { α, α, α, α, α, α } mul({ β, β, β, β, β, β } m) {
      { Matrix, Vector<Vector<Integer>>, Vector<_?Vector<Integer>>,
       Vector<_?Vector<_?Integer>>, Vector<_?Vector<^?Integer>>,
       Vector<^?Vector<Integer>> } ret = new Matrix();
      int i = 0; while(i <size()) {
         { ι, ι, ι, ι, ι, ι } v1 = this.elementAt(i);
         { κ, κ, κ, κ, κ, κ } v2 = new Vector<Integer>();
         int j = 0; while(j < size()) {
            { χ, χ, χ, χ, χ, χ } erg = 0;
            int k = 0; while(k < v1.size()) {
               erg = erg +({ ξ,ξ,ξ,ξ,ξ,ξ }({ ι,ι,ι,ι,ι,ι } v1).elementAt(k))
                * ({ ψ,ψ,ψ,ψ,ψ,ψ }
                    ({ ϕ,ϕ,ϕ,ϕ,ϕ,ϕ }
                     ({ β,β,β,β,β,β } m).elementAt(k)).elementAt(j)); k++;}
            v2.addElement({ χ, χ, χ, χ, χ, χ } erg); j++; }
         ret.addElement({ μ, μ, μ, μ, μ, μ } v2); i++; }
      return ret; }}
```

v1 = this.elementAt(i);

```
{α} mul({β} m) {
    ...
    {ι} v1 = ({Matrix} this).elementAt(i);
    ...
}
```

Unification: $\texttt{Matrix} \lessdot \texttt{Vector<}\iota\texttt{>}$

$\Rightarrow$

$$\iota = \texttt{Vector<Integer>}$$
$$\iota = \texttt{Vector<}_?\texttt{Integer>}$$
$$\iota = \texttt{Vector<}^?\texttt{Integer>}$$

return ret;

```
{ α } mul({ β } m) {
    ...
    return { γ } ret;
}
```

Unification: $\gamma \lessdot \alpha$ for

$\gamma = \texttt{Matrix}$

$\gamma = \texttt{Vector<Vector<Integer>>}$

$\gamma = \texttt{Vector<}^{?}\texttt{Vector<Integer>>}$

Result:

$\alpha = \texttt{Matrix}$

$\alpha = \texttt{Vector<Vector<Integer>>}$

$\alpha = \texttt{Vector<}^{?}\texttt{Vector<Integer>>}$

$\alpha = \texttt{Vector<}_{?}\texttt{Vector<Integer>>}$

$\alpha = \texttt{Vector<}_{?}\texttt{Vector<}_{?}\texttt{Integer>>}$

$\alpha = \texttt{Vector<}_{?}\texttt{Vector<}^{?}\texttt{Integer>>}$

Result:

$$\texttt{mul}: \&_{\beta,\alpha}(\beta \rightarrow \alpha),$$

where

$\beta \leq^* \texttt{Vector<}_?\texttt{Vector<}_?\texttt{Integer>>}$,
$\texttt{Matrix} \leq^* \alpha$

Implementation

- Overloading–Example
- Return–Example
- Matrix–Example

Implementation

- Overloading–Example
- Return–Example
- Matrix–Example

Purpuse: Byte-code generation for methods with intersection types

Code generation for method with intersection types[5]

- Byte-code allows no intersection types
- First approach: generate for each element of the intersection type an own method

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[5] M. Pluemicke, *Intersection types in java*. PPPJ 2008.

Example: class `OL` I

```
class OL {
     Integer m(x) { return x + x; }   //Integer → Integer
     Boolean m(x) { return x || x; }  //Boolean → Boolean
}

class Main {
    main(x) {    //   Integer → Integer & Boolean → Boolean
       ol;
       ol = new OL();
       return ol.m(x);
    }
}
```

Example: class OL II

Result for `Main`:

```
class Main {
    Integer main(Integer x) {
        OL ol;
        ol = new OL();
        return ol.m(x); }

    Boolean main(Boolean x) {
        OL ol;
        ol = new OL();
        return ol.m(x);
    }
}
```

Example: class `OL` II

Result for `Main`:

```
class Main {
    Integer main(Integer x) {
        OL ol;
        ol = new OL();
        return ol.m(x); }

    Boolean main(Boolean x) {
        OL ol;
        ol = new OL();
        return ol.m(x);
    }
}
```

Java program is correct

Example: Multiplication of matrices

$$\texttt{mul:}\ \&_{\beta,\alpha}(\beta\rightarrow\alpha),$$

where

$\beta \leq^{*}$ `Vector<? extends Vector<? extends Integer>>`,
`Matrix` $\leq^{*} \alpha$

Example: Multiplication of matrices

$$\texttt{mul:}\ \&_{\beta,\alpha}(\beta\longrightarrow\alpha),$$

where

$\beta \leq^* \texttt{Vector<? extends Vector<? extends Integer>>}$,
$\texttt{Matrix} \leq^* \alpha$

```
class Matrix extends Vector<Vector<Integer>> {
  Matrix mul(Vector<? extends Vector<? extends Integer>> m) { ... }
  Matrix mul(Vector<? extends Vector<Integer>> m) { ... }
  Matrix mul(Vector<Vector<Integer>> m) { ... }
  ...
  Vector<Vector<Integer>> mul(Vector<Vector<Integer>> m) { ... }
  ...
  Vector<? extends Vector<? extends Integer>> mul(Matrix m) { ... }
```

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  ...
  Vector<? extends Vector<? extends Integer>> mul(Matrix m) { ... }
```

Not a correct Java program

Group elements of the intersection type

Idea:

1. Group all elements which
   - executes the same code
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2. Generate new methods only for the groups

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Callgraph of the method m in the class cl with the typing τ.

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Subtype of function types: Subtyping ordering
$\theta_i \leq^* \theta'_i, \theta \leq^* \theta' \Rightarrow$

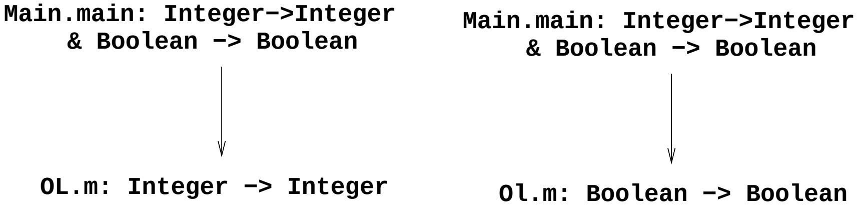
$$\theta'_1 \times \ldots \times \theta'_n \rightarrow \theta \leq^* \theta_1 \times \ldots \times \theta_n \rightarrow \theta'$$

Example class OL I

Callgraph

$\mathcal{CG}(\,\texttt{Main.main} : \texttt{Integer} \rightarrow \texttt{Integer}\,)$ $=$

$\mathcal{CG}(\,\texttt{Main.main} : \texttt{Boolean} \rightarrow \texttt{Boolean}\,)$ $=$



`Main.main: Integer->Integer & Boolean -> Boolean`

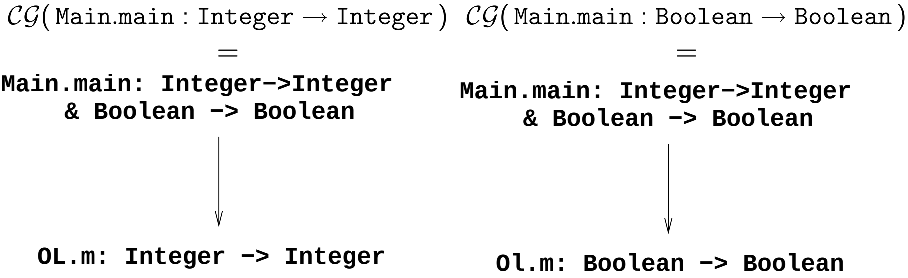
`OL.m: Integer -> Integer`

`Main.main: Integer->Integer & Boolean -> Boolean`

`Ol.m: Boolean -> Boolean`

Example class OL I

Callgraph



$\mathcal{CG}(\,\texttt{Main.main} : \texttt{Integer} \to \texttt{Integer}\,)$

$=$

Main.main: Integer->Integer & Boolean -> Boolean

OL.m: Integer -> Integer

$\mathcal{CG}(\,\texttt{Main.main} : \texttt{Boolean} \to \texttt{Boolean}\,)$

$=$

Main.main: Integer->Integer & Boolean -> Boolean

Ol.m: Boolean -> Boolean

Subtype

$\texttt{Integer} \to \texttt{Integer}$

$\texttt{Boolean} \to \texttt{Boolean}$

Example class OL II

Code generation

```
class Main {
    Integer main(Integer x) {
        OL ol;
        ol = new OL();
        return ol.m(x); }

    Boolean main(Boolean x) {
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        ol = new OL();
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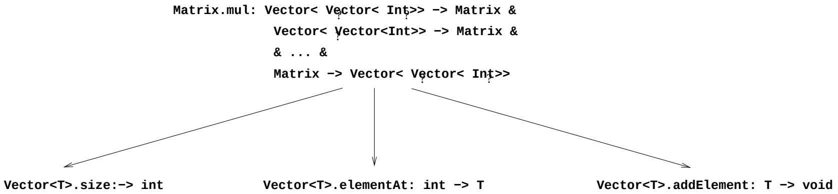
    } }
```

Code is unchanged in comparison to the first approach

Example class `Matrix` I

Callgraph $\mathcal{CG}($ `Matrix.mul` : τ $)$ for all τ



```
Matrix.mul: Vector< Vector< Int>> -> Matrix &
            Vector< Vector<Int>> -> Matrix &
            & ... &
            Matrix -> Vector< Vector< Int>>
```

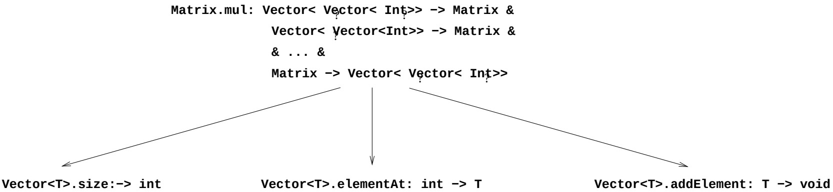
`Vector<T>.size:-> int`

`Vector<T>.elementAt: int -> T`

`Vector<T>.addElement: T -> void`

Example class `Matrix` I

Callgraph $\mathcal{CG}($ `Matrix.mul` $: \tau\,)$ for all τ



```
Matrix.mul: Vector< ?Vector< ?Int>> -> Matrix &
            Vector< ?Vector<Int>> -> Matrix &
            & ... &
            Matrix -> Vector< ?Vector< ?Int>>

Vector<T>.size:-> int      Vector<T>.elementAt: int -> T      Vector<T>.addElement: T -> void
```

Subtype:

$$\texttt{Vector<? extends Vector<? extends Integer>>} \longrightarrow \texttt{Matrix}$$

Example class Matrix II

Code generation (only one method!)

```
Matrix mul(Vector<? extends Vector<? extends Integer>> m) {
   Matrix ret = new Matrix();
   int i = 0;
   while(i <size()) {
       Vector<Integer> v1 = this.elementAt(i);
       Vector<Integer> v2 = new Vector<Integer>();
       int j = 0;
       while(j < size()) {
           int erg = 0;
           int k = 0;
           while(k < v1.size()) { erg = erg + ...; k++; }
           v2.addElement(new Integer(erg)); j++; }
       ret.addElement(v2); i++; }
   return ret; }}
```

The algorithm

Input: A Java program p with inferred (intersection) types.

Output: A Java program p', where the methods have standard Java types. The semantics of p and p' are equal.

1. Step: For every class cl in p consider for each method m the intersection type ty_m:
   - Build the callgraph $\mathcal{CG}(\, cl.m : \tau\,)$ for each function type τ of the intersection type ty_m.
   - Group all elements τ of ty_m, where $\mathcal{CG}(\, cl.m : \tau\,)$ is the same graph and there is a subtype.

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2. Step: Determine the subtype of the respective group.
3. Step: Generate for each group of function types the corresponding Java code with the subtype as standard typing in p'.

Principal Typing

Definition [Damas, Milner 1982]:

“A type-scheme for a declaration is a *principal type-scheme*, if any other type-scheme for the declaration is a generic instance of it.”

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Generalized definition for Java:

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Generalized definition for Java:

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This refined definition guarantees, that for each method, which is generated by the resolving algorithm, at least one typing is contained in the principal type.

Conclusion of Java 5 type inference

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- Resolving algorithm of intersection types, which allows to generate byte code for intersection types
- Principal type property.

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- Principal type property.

Outlook

At the moment: Type inference algorithm infers typings, which are later erased as supertypes by the resolving algorithm.

Purpuse: Type inference algorithm infers only subtypes, such that no typings are erased in the resolving algorithm.

Implementation: The resolving algorithm and optimize the type inference algorithm

Closures (λ–expressions) in Java 7?

Motivation: Bulk-data APIs like parallel arrays

- parallelism approach: sorting, searching, selection

Example:

```
public class Student {
    String name;
    int graduationYear;
    double gpa; //grade point average
}

ParallelArray<Student> students
    = new ParallelArray<Student>(fjPool, data);

double bestGpa = students.withFilter(isSenior)
                         .withMapping(selectGpa)
                         .max();
```

Realization by helper objects

```
static final Ops.Predicate<Student> isSenior
    = new Ops.Predicate<Student>() {
        public boolean op(Student s) {
            return s.graduationYear == Student.THIS_YEAR;
        }
    };

static final Ops.ObjectToDouble<Student> selectGpa
    = new Ops.ObjectToDouble<Student>() {
        public double op(Student student) {
            return student.gpa;
        }
    };
```

Realization by closures (λ–expressions)

```
double bestGpa
    = students.withFilter(
                  #(Student s)(s.graduationYear == THIS_YEAR))
              .withMapping(#(Student s)(s.gpa))
              .max();
```

Different approaches

- Closures for the Java Programming Language: BGGA
  [Bracha, Gafter, Gosling, von der Ahé]
- Concise Instance Creation Expressions: Closures without Complexity: CICE
  [Lee, Lea, Bloch]
- First-class methods: Java-style closures: FCM
  [Colebourne, Schulz]

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[6] http://mail.openjdk.java.net/pipermail/lambda-dev/attachments/20100212/af8d2cc5/attachment-0001.txt

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- First-class methods: Java-style closures: FCM
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Our approach is following:
Project Lambda[6] Java Language Specification draft (Version 0.1.5)

---

[6] http://mail.openjdk.java.net/pipermail/lambda-dev/attachments/20100212/af8d2cc5/attachment-0001.txt

Mark Reinhold's Blog (Principal Engineer Java SE and OpenJDK)

Two key features are needed:

- A literal syntax, for writing closures, and
- Function types, so that closures are first-class citizens in the type system.

To integrate closures with the rest of the language and the platform we need two additional features:

- Closure conversion to implement a single-method interface or abstract class and
- Extension methods

The language Java_λ

$$\begin{array}{lcl}
\mathit{Source} & := & \mathit{class}* \\
\mathit{class} & := & \mathrm{Class}(\mathit{stype},[\ \mathrm{extends}(\ \mathit{stype}\ ),]\mathit{IVarDecl}*, \mathit{FunDecl}*) \\
\mathit{IVarDecl} & := & \mathrm{InstVarDecl}(\ \mathit{stype}, \mathit{var}\ ) \\
\mathit{FunDecl} & := & \mathrm{Fun}(\ \mathit{fname}, \underline{[\mathit{type}]}, \mathit{lambdaexpr}\ ) \\
\mathit{block} & := & \mathrm{Block}(\ \mathit{stmt}*\ ) \\
\mathit{stmt} & := & \mathit{block}\ \mid\ \mathrm{Return}(\ \mathit{expr}\ )\ \mid\ \mathrm{While}(\ \mathit{bexpr}, \mathit{block}\ ) \\
 & \mid & \mathrm{LocalVarDecl}(\ \mathit{var}\underline{[, \mathit{type}]}\ )\ \mid \mathrm{If}(\ \mathit{bexpr}, \mathit{block}[, \mathit{block}]\ ) \\
 & \mid & \mathit{stmtexpr} \\
\mathit{lambdaexpr} & := & \mathrm{Lambda}(\ ((\mathit{var}\underline{[, \mathit{type}])})*, (\mathit{stmt}\ \mid\ \mathit{expr})\ ) \\
\mathit{stmtexpr} & := & \mathrm{Assign}(\ \mathit{var}, \mathit{expr}\ )\ \mid\ \mathrm{New}(\ \mathit{stype}, \mathit{expr}*\ ) \\
 & \mid & \mathrm{Eval}(\ \mathit{expr}, \mathit{expr}*\ ) \\
\mathit{expr} & := & \mathit{lambdaexpr}\ \mid\ \mathit{stmtexpr}\ \mid\ \mathrm{this}\ \mid\ \mathrm{super} \\
 & \mid & \mathrm{LocalOrFieldVar}(\ \mathit{var}\ )\ \mid\ \mathrm{InstVar}(\ \mathit{expr}, \mathit{var}\ ) \\
 & \mid & \mathrm{InstFun}(\ \mathit{expr}, \mathit{fname}\ )\ \mid\ \mathit{bexp}\ \mid\ \mathit{sexp}
\end{array}$$

Types $\mathrm{Type}_{TS}(\,BTV\,)$

- $\mathrm{SType}_{TS}(\,BTV\,) \subseteq \mathrm{Type}_{TS}(\,BTV\,)$
- For $\theta_i \in \mathrm{Type}_{TS}(\,BTV\,)$

$$\#\,\theta_0\,(\theta_1, \ldots, \theta_n) \in \mathrm{Type}_{TS}(\,BTV\,)$$

(*straw-man syntax*, correspond to $\theta_1 \times \ldots \times \theta_n \rightarrow \theta_0$)

Subtyping–relation

Let $\leq^*$ be the Java subtyping relation on simple types $\mathrm{SType}_{TS}(\, BTV \,)$.

The continuation on $\mathrm{Type}_{TS}(\, BTV \,)$ is defined as:

$$\#\,\theta_0\,(\theta'_1, \ldots, \theta'_n) \leq^* \#\,\theta'_0\,(\theta_1, \ldots, \theta_n) \quad \textit{iff} \quad \theta_i \leq^* \theta'_i.$$

λ–Abstraction

[lambda$_{\textbf{stmt}}$]

$$\frac{(O \cup \{\, x_i : \theta_i \,\}, \tau, \tau') \ \triangleright_{Stmt} \ s : \theta}{(O, \tau, \tau') \ \triangleright_{Expr} \ \mathrm{Lambda}(\,(x_1, \ldots, x_n), s\,) : \#\,\theta\,(\theta_1, \ldots, \theta_n)}$$

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[lambda$_{\text{expr}}$]

$$\frac{(O \cup \{\, x_i : \theta_i \,\}, \tau, \tau') \ \triangleright_{Expr} \ e : \theta}{(O, \tau, \tau') \ \triangleright_{Expr} \ \text{Lambda}(\,(x_1, \ldots, x_n), e\,) : \#\,\theta\,(\theta_1, \ldots, \theta_n)}$$

Application

[App]

$$\frac{(O, \tau, \tau') \rhd_{Expr} \; e : \#\,\theta\,(\theta'_1, \ldots, \theta'_n), \quad (O, \tau, \tau') \rhd_{Expr} \; e_i : \theta_i}{(O, \tau, \tau') \rhd_{Expr} \; \mathrm{Eval}(\, e, e_1 \ldots e_n \,) : \theta} \; \theta_i \leq^* \theta'_i$$

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[InstFun]

$$\frac{(O, \tau, \tau') \rhd_{Expr} \; re : \overline{\theta}, \qquad O_{\overline{\theta}} \rhd_{Id} \; f : \#\,\theta\,(\theta_1, \ldots, \theta_n)}{(O, \tau, \tau') \rhd_{Expr} \; \text{InstFun}(\, re, f \,) : \#\,\theta\,(\theta_1, \ldots, \theta_n)}$$

Adapt Fuh and Mishra's algorithm[7].

- Java$_\lambda$ type system is equivalent
- subtyping, but
- no overloading

---

[7] Y.-C. Fuh, P. Mishra. Type inference with subtypes. ESOP '88

Adapt Fuh and Mishra's algorithm[7].

- Java$_\lambda$ type system is equivalent
- subtyping, but
- no overloading
- **Problem**: Fuh and Mishra's algorithm determines *well typings* (set of possibly not unique solvable unequations)

---

[7] Y.-C. Fuh, P. Mishra. Type inference with subtypes. ESOP '88

Fuh und Mishra's algorithm

$$\textbf{WTYPE} : \texttt{TypeAssumptions} \times \texttt{Expression} \rightarrow \texttt{WellTyping} + \{\, \mathit{fail} \,\}$$

Fuh und Mishra's algorithm

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TYPE: $\texttt{TypeAssump} \times \texttt{Expr} \rightarrow \texttt{Type} \times \texttt{CoercionSet}$

Construct the set of coercions by running over the Expression.

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Extended unification to adopt the structure of the coercions.

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Eliminate type constructors, especially the *functor* and the *tuple-construction*

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Eliminate type constructors, especially the *functor* and the *tuple-construction*

CONSISTENT : $\texttt{AtomicCoercionSet} \rightarrow \texttt{Boolean} + \{ \mathit{fail} \}$

Consistence check, and additionally determination of possible solutions.

Adaption to the Java$_\lambda$ type system

SIMPLIFY: The [Martelli, Montanari 1982] unification is substituted by the [Pluemicke 2009] type unification[8]

CONSISTENCE: The functions *above* and *below* are substituted by the functions `greater` and `smaller`[8], as *above* and *below* are not finite in the Java$_\lambda$ type system.

---

[8] M. Pluemicke. *Java type unification with wildcards*, INAP 07. LNAI 5437.

Example

```
class Matrix extends Vector<Vector<Integer>> {
  //Matrix -> ((Matrix, Matrix) -> Matrix) -> Matrix
  ##Matrix(#Matrix(Matrix, Matrix))(Matrix)
    op = #(Matrix m)(#(#Matrix(Matrix, Matrix) f)(f(this, m)))
  }
```

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  //Matrix -> ((Matrix, Matrix) -> Matrix) -> Matrix
  ##Matrix(#Matrix(Matrix, Matrix))(Matrix)
    op = #(Matrix m)(#(#Matrix(Matrix, Matrix) f)(f(this, m)))
  }
  ...
  public static void main(String[] args) {
      Matrix m1 = new Matrix(...);
      Matrix m2 = new Matrix(...);
      m1.op.(m2).(#(Matrix m1, Matrix m2) {
                      Matrix ret = new Matrix ();
                      ⋮ //matrice multiplication
                      return ret;
                      })
  } }
```

Goal

Typed syntax:

```
##Matrix(#Matrix(Matrix, Matrix))(Matrix)
  op = #(Matrix m)(#(#Matrix(Matrix, Matrix) f)(f(this, m)))
```

Goal

Typed syntax:

```
##Matrix(#Matrix(Matrix, Matrix))(Matrix)
  op = #(Matrix m)(#(#Matrix(Matrix, Matrix) f)(f(this, m)))
```

Typeless syntax:

```
  op = #(m)(#(f)(f(this, m)))
```

Goal: The system determines the type

```
##Matrix(#Matrix(Matrix, Matrix))(Matrix)
```

automatically.

$WTYPE(\,\emptyset, \#(\mathtt{m})(\#(\mathtt{f})(\mathtt{f}.(\mathtt{this}, \mathtt{m})))\,),$ (TYPE)

- $t_m \longrightarrow t_{\#f} \lessdot t_{\mathrm{op}}$
- $t_f \longrightarrow t_{f(\,this, m\,)} \lessdot t_{\#f}$
- $t_f \lessdot (t_1, t_2) \longrightarrow t_3$
- $\mathtt{Matrix} \lessdot t_1$
- $t_m \lessdot t_2$
- $t_3 \lessdot t_{f(\,this, m\,)}$

$WTYPE(\,\emptyset, \#(\mathtt{m})(\#(\mathtt{f})(\mathtt{f}.(\mathtt{this}, \mathtt{m})))\,), \qquad (\mathrm{MATCH})$

- $t_m \to ((\epsilon_2, \epsilon'_2) \to \epsilon''_2) \to \gamma'_1 \lessdot \beta \to ((\epsilon_3, \epsilon'_3) \to \epsilon''_3) \to \gamma'_2,$

  $(t_{op} \mapsto \beta \to \beta'), (t_{\#f} \mapsto \gamma_1 \to \gamma'_1), (\beta' \mapsto \gamma_2 \to \gamma'_2),$
  $(\gamma_1 \mapsto (\epsilon_2, \epsilon'_2) \to \epsilon''_2), (\gamma_2 \mapsto (\epsilon_3, \epsilon'_3) \to \epsilon''_3)$

- $((\epsilon_1, \epsilon'_1) \to \epsilon''_1) \to t_{f(\,this, m\,)} \lessdot ((\epsilon_2, \epsilon'_2) \to \epsilon''_2) \to \gamma'_1$

  $(t_{\#f} \mapsto \gamma_1 \to \gamma'_1), (t_f \mapsto (\epsilon_1, \epsilon'_1) \to \epsilon''_1), (\gamma_1 \mapsto (\epsilon_2, \epsilon'_2) \to \epsilon''_2)$

- $(\epsilon_1, \epsilon'_1) \to \epsilon''_1 \lessdot (t_1, t_2) \to t_3$

  $(t_f \mapsto (\epsilon_1, \epsilon'_1) \to \epsilon''_1), (\gamma_1 \mapsto (\epsilon_2, \epsilon'_2) \to \epsilon''_2)$

- $\mathtt{Matrix} \lessdot t_1$
- $t_m \lessdot t_2$
- $t_3 \lessdot t_{f(\,this, m\,)}$

$WTYPE(\,\emptyset, \#(\mathtt{m})(\#(\mathtt{f})(\mathtt{f}.(\mathtt{this}, \mathtt{m})))\,)$, (SIMPLIFY)

- $\beta \lessdot t_m$,
  $\epsilon_2 \lessdot \epsilon_3,\ \epsilon'_2 \lessdot \epsilon'_3,\ \epsilon''_3 \lessdot \epsilon''_2$,
  $\gamma'_1 \lessdot \gamma'_2$
- $\epsilon_1 \lessdot \epsilon_2,\ \epsilon'_1 \lessdot \epsilon'_2,\ \epsilon''_2 \lessdot \epsilon''_1$,
  $t_{f(\,this,m\,)} \lessdot \gamma'_1$
- $t_1 \lessdot \epsilon_1$,
  $t_2 \lessdot \epsilon'_1$,
  $\epsilon''_1 \lessdot t_3$
- $\mathtt{Matrix} \lessdot t_1$
- $t_m \lessdot t_2$
- $t_3 \lessdot t_{f(\,this,m\,)}$

$WTYPE(\,\emptyset, \#(\texttt{m})(\#(\texttt{f})(\texttt{f}.(\texttt{this}, \texttt{m})))\,),\ (\mathrm{CONSISTENCE})$

It	$Coercion$	$l_{\texttt{Matrix}}$	l_{t_1}	l_{ϵ_1}	l_{ϵ_2}	l_{ϵ_3}	...
0		M	$*$	$*$	$*$	$*$	
1	$\texttt{M} \lessdot t_1$	M	M, V<V<Int>>	$*$	$*$	$*$	
1	$t_1 \lessdot \epsilon_1$	M	M, V<V<Int>>	M, V<V<Int>>	$*$	$*$	
1	$\epsilon_1 \lessdot \epsilon_2$	M	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	$*$	
1	$\epsilon_2 \lessdot \epsilon_3$	M	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	
1	...	M	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	
2	...	M	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	M, V<V<Int>>	

Result

$$\mathrm{op} : \beta \rightarrow ((\epsilon_3, \epsilon'_3) \rightarrow \epsilon''_3) \rightarrow \gamma'_2$$

with

$\epsilon_3 = \texttt{Matrix}, \texttt{Vector<Vector<Integer>>}$

$\beta \lessdot t_m \lessdot t_2 \lessdot \epsilon'_1 \lessdot \epsilon'_2 \lessdot \epsilon'_3$

$\epsilon''_3 \lessdot \epsilon''_2 \lessdot \epsilon''_1 \lessdot t_3 \lessdot t_{f(\,this, m\,)} \lessdot \gamma'_1 \lessdot \gamma'_2$

Result

$$\text{op} : \beta \rightarrow ((\epsilon_3, \epsilon_3') \rightarrow \epsilon_3'') \rightarrow \gamma_2'$$

with

$\epsilon_3 = \texttt{Matrix}, \texttt{Vector<Vector<Integer>>}$

$\beta \lessdot t_m \lessdot t_2 \lessdot \epsilon_1' \lessdot \epsilon_2' \lessdot \epsilon_3'$

$\epsilon_3'' \lessdot \epsilon_2'' \lessdot \epsilon_1'' \lessdot t_3 \lessdot t_{f(\,this, m\,)} \lessdot \gamma_1' \lessdot \gamma_2'$

This is a well-typing.

Compare to the goal:

$\texttt{Matrix} \rightarrow ((\texttt{Matrix}, \texttt{Matrix}) \rightarrow \texttt{Matrix}) \rightarrow \texttt{Matrix}$

The well-typing is more principal.

Conclusion and future work

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Future work

- Improve the type inference algorithm for Java 5, such that only principal types are inferred
- Integrate *well typings* into Java 7
- Implementation:
  - Code-generation for intersection types
  - Adopted Fuh and Mishra algorithm